Ch11 Hypothesis Testing AP Exam Free Response Questions:

$$\Pr(\text{Type I error}) = \Pr(\text{Reject } H_0 | \, H_0 \text{ is true}) = \alpha.$$

However, in general, the probability of making Type II error,

$$\Pr(\text{Type II error}) = \Pr(\text{Not Reject } H_0 | \, H_0 \text{ is false}),$$

is different across different test statistics. The power of test is defined as

$$\text{Power} = 1\text{-}\Pr(\text{Type II error}) = 1\text{-}\Pr(\text{Not Reject } H_0 | \, H_0 \text{ is false}).$$

2015 #4

A researcher conducted a medical study to investigate whether taking a low-dose aspirin reduces the chance of developing colon cancer. As part of the study, 1,000 adult volunteers were randomly assigned to one of two groups. Half of the volunteers were assigned to the experimental group that took a low-dose aspirin each day, and the other half were assigned to the control group that took a placebo each day. At the end of six years, 15 of the people who took the low-dose aspirin had developed colon cancer and 26 of the people who took the placebo had developed colon cancer. At the significance level $\alpha = 0.05$, do the data provide convincing statistical evidence that taking a low-dose aspirin each day would reduce the chance of developing colon cancer among all people similar to the volunteers?

2021 #4

4. The manager of a large company that sells pet supplies online wants to increase sales by encouraging repeat purchases. The manager believes that if past customers are offered \$10 off their next purchase, more than 40 percent of them will place an order. To investigate the belief, 90 customers who placed an order in the past year are selected at random. Each of the selected customers is sent an e-mail with a coupon for \$10 off the next purchase if the order is placed within 30 days. Of those who receive the coupon, 38 place an order.

(a) Is there convincing statistical evidence, at the significance level of $\alpha = 0.05$, that the manager's belief is correct? Complete the appropriate inference procedure to support your answer.

(b) Based on your conclusion from part (a), which of the two errors, Type I or Type II, could have been made? Interpret the consequence of the error in context.

2019 #4

4. Tumbleweed, commonly found in the western United States, is the dried structure of certain plants that are blown by the wind. Kochia, a type of plant that turns into tumbleweed at the end of the summer, is a problem for farmers because it takes nutrients away from soil that would otherwise go to more beneficial plants. Scientists are concerned that kochia plants are becoming resistant to the most commonly used herbicide, glyphosate. In 2014, 19.7 percent of 61 randomly selected kochia plants were resistant to glyphosate. In 2017, 38.5 percent of 52 randomly selected kochia plants were resistant to glyphosate. Do the data provide convincing statistical evidence, at the level of $\alpha = 0.05$, that there has been an increase in the proportion of all kochia plants that are resistant to glyphosate?

2018 #6

6. Systolic blood pressure is the amount of pressure that blood exerts on blood vessels while the heart is beating. The mean systolic blood pressure for people in the United States is reported to be 122 millimeters of mercury (mmHg) with a standard deviation of 15 mmHg.

The wellness department of a large corporation is investigating whether the mean systolic blood pressure of its employees is greater than the reported national mean. A random sample of 100 employees will be selected, the systolic blood pressure of each employee in the sample will be measured, and the sample mean will be calculated.

Let μ represent the mean systolic blood pressure of all employees at the corporation. Consider the following hypotheses.

$$H_0 : \mu = 122$$
$$H_a : \mu > 122$$

(a) Describe a Type II error in the context of the hypothesis test.

(b) Assume that σ, the standard deviation of the systolic blood pressure of all employees at the corporation, is 15 mmHg. If $\mu = 122$, the sampling distribution of $\bar{x}$ for samples of size 100 is approximately normal with a mean of 122 mmHg and a standard deviation of 1.5 mmHg. What values of the sample mean $\bar{x}$ would represent sufficient evidence to reject the null hypothesis at the significance level of $\alpha = 0.05$?

The actual mean systolic blood pressure of all employees at the corporation is 125 mmHg, not the hypothesized value of 122 mmHg, and the standard deviation is 15 mmHg.

(c) Using the actual mean of 125 mmHg and the results from part (b), determine the probability that the null hypothesis will be rejected.

(d) What statistical term is used for the probability found in part (c) ?

(e) Suppose the size of the sample of employees to be selected is greater than 100. Would the probability of rejecting the null hypothesis be greater than, less than, or equal to the probability calculated in part (c) ? Explain your reasoning.

2011 #4

4. High cholesterol levels in people can be reduced by exercise, diet, and medication. Twenty middle-aged males with cholesterol readings between 220 and 240 milligrams per deciliter (mg/dL) of blood were randomly selected from the population of such male patients at a large local hospital. Ten of the 20 males were randomly assigned to group A, advised on appropriate exercise and diet, and also received a placebo. The other 10 males were assigned to group B, received the same advice on appropriate exercise and diet, but received a drug intended to reduce cholesterol instead of a placebo. After three months, posttreatment cholesterol readings were taken for all 20 males and compared to pretreatment cholesterol readings. The tables below give the reduction in cholesterol level (pretreatment reading minus posttreatment reading) for each male in the study.

Group A (placebo)

Reduction (in mg/dL)	2	19	8	4	12	8	17	7	24	1

Mean Reduction: 10.20 Standard Deviation of Reductions: 7.66

Group B (cholesterol drug)

Reduction (in mg/dL)	30	19	18	17	20	−4	23	10	9	22

Mean Reduction: 16.40 Standard Deviation of Reductions: 9.40

Do the data provide convincing evidence, at the $\alpha = 0.01$ level, that the cholesterol drug is effective in producing a reduction in mean cholesterol level beyond that produced by exercise and diet?

2010 #6

6. Hurricane damage amounts, in millions of dollars per acre, were estimated from insurance records for major hurricanes for the past three decades. A stratified random sample of five locations (based on categories of distance from the coast) was selected from each of three coastal regions in the southeastern United States. The three regions were Gulf Coast (Alabama, Louisiana, Mississippi), Florida, and Lower Atlantic (Georgia, South Carolina, North Carolina). Damage amounts in millions of dollars per acre, adjusted for inflation, are shown in the table below.

HURRICANE DAMAGE AMOUNTS IN MILLIONS OF DOLLARS PER ACRE

	Distance from Coast				
	< 1 mile	1 to 2 miles	2 to 5 miles	5 to 10 miles	10 to 20 miles
Gulf Coast	24.7	21.0	12.0	7.3	1.7
Florida	35.1	31.7	20.7	6.4	3.0
Lower Atlantic	21.8	15.7	12.6	1.2	0.3

(a) Sketch a graphical display that compares the hurricane damage amounts per acre for the three different coastal regions (Gulf Coast, Florida, and Lower Atlantic) and that also shows how the damage amounts vary with distance from the coast.

(b) Describe differences and similarities in the hurricane damage amounts among the three regions.

Because the distributions of hurricane damage amounts are often skewed, statisticians frequently use rank values to analyze such data.

(c) In the table below, the hurricane damage amounts have been replaced by the ranks 1, 2, or 3. For each of the distance categories, the highest damage amount is assigned a rank of 1 and the lowest damage amount is assigned a rank of 3. Determine the missing ranks for the 10-to-20-miles distance category and calculate the average rank for each of the three regions. Place the values in the table below.

ASSIGNED RANKS WITHIN DISTANCE CATEGORIES

	Distance from Coast					
	< 1 mile	1 to 2 miles	2 to 5 miles	5 to 10 miles	10 to 20 miles	Average Rank
Gulf Coast	2	2	3	1		
Florida	1	1	1	2		
Lower Atlantic	3	3	2	3		

(d) Consider testing the following hypotheses.

H_0: There is no difference in the distributions of hurricane damage amounts among the three regions.

H_a: There is a difference in the distributions of hurricane damage amounts among the three regions.

If there is no difference in the distribution of hurricane damage amounts among the three regions (Gulf Coast, Florida, and Lower Atlantic), the expected value of the average rank for each of the three regions is 2. Therefore, the following test statistic can be used to evaluate the hypotheses above:

$$Q = 5\left[\left(\bar{R}_G - 2\right)^2 + \left(\bar{R}_F - 2\right)^2 + \left(\bar{R}_A - 2\right)^2\right]$$

where $\bar{R}_G$ is the average rank over the five distance categories for the Gulf Coast (and $\bar{R}_F$ and $\bar{R}_A$ are similarly defined for the Florida and Lower Atlantic coastal regions).

Calculate the value of the test statistic Q using the average ranks you obtained in part (c).